Written Exam for the M.Sc. in Economics Summer 2012
Fixed Income Derivatives
Final Exam - Master's Course

21st-23rd of April 2012

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.
If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.
The paper must be uploaded as <u>one PDF document</u> (including the standard cover and the appendices). The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Absalon.

Focus on Exam Cheating

In case of presumed exam cheating, which is observed by either the examination registration of the respective study programmes, the invigilation or the course lecturer, the Head of Studies will make a preliminary inquiry into the matter, requesting a statement from the course lecturer and possibly the invigilation, too. Furthermore, the Head of Studies will interview the student. If the Head of Studies finds that there are reasonable grounds to suspect exam cheating, the issue will be reported to the Rector. In the course of the study and during examinations, the student is expected to conform to the rules and regulations governing academic integrity. Academic dishonesty includes falsification, plagiarism, failure to disclose information, and any other kind of misrepresentation of the student's own performance and results or assisting another student herewith. For example failure to indicate sources in written assignments is regarded as failure to disclose information. Attempts to cheat at examinations are dealt with in the same manner as exam cheating which has been carried through. In case of exam cheating, the following sanctions may be imposed by the Rector:

1. A warning
2. Expulsion from the examination
3. Suspension from the University for at limited period or permanent expulsion.

The Faculty of Social Sciences
The Study and Examination Office
October 2006

Introduction

The exam questions are accompanied by a spreadsheet with data that can be downloaded from the course website. Also, you need to download the latest version of `fidAnalytics.xla`. Make sure to read all the questions as well as the data spreadsheet carefully. The purpose of the exam is to test your ability to price and risk manage derivatives. In order to receive the grade 12, you must demonstrate the ability to implement and use the models and methodologies covered in the course.

GOOD LUCK!

Formalities

Do NOT include your name or any other personal details in your solution. The deadline for uploading your solution is Monday, April 23rd at 10.00. Late solutions will not be accepted.

If you believe that you have found any errors or unclarities in the assignment, the lecturer can be reached via e-mail at martin.linderstroem@gmail.com. You are not allowed to discuss the exam with anyone other than the lecturer.

Excel/VBA settings

In order to avoid any differences in results stemming from numerical issue alone, you should set your Excel `SOLVER` to have the following settings:

- Max Time=300
- Iterations=300
- Precision=0.000001
- Tolerance=5%
- Convergence=0.000001
- Automatic Scaling=Yes

1 The Swap Curve

1.1 Introducing money market basis

One of the consequences of the financial crisis was a substantial widening in the so-called money market basis. Money market basis is the difference in credit risk embedded in xIBOR fixings of different tenors. The presence of money market basis means that you cannot replicate a e.g. 6M forward xIBOR rate by taking positions in the corresponding two 3M forward rates. Disregard discounting, explain why the following market observations exhibit money market basis:

- 3M EURIBOR=0.753%
- 3x6 EUR FRA=0.923%
- 6M EURIBOR=1.048%

1.2 Pricing money market basis swaps in VBA

Following the financial crisis, an active market for so called money market basis swaps has arisen. One product traded in this market, is the float-float basis swap. In such a contract, you swap a xIBOR fixing of a short tenor (e.g. 3M EURIBOR) against a xIBOR fixing of a longer tenor (e.g. 6M EURIBOR). Since the shorter tenor xIBOR stream will typically be less valuable compared to the longer tenor ditto, you add a spread to the shorter xIBOR tenor such that the swap has zero value at inception. This spread follows the conventions of the shorter tenor xIBOR leg in terms of frequency and day count (e.g. 3M frequency and Act/360 for a 3M EURIBOR leg).

Build a VBA function called `fidMMBasisSpread` that can price such a contract. That is, you should write a function that can take two separate forward curve inputs — one dedicated each tenor. Use the function `fidFloatingPv` as a starting point, but augment the function with the specific inputs:

- `ShortTenor` As `String`
- `LongTenor` As `String`
- `ShortFwdCurveMat` As `Variant`
- `ShortFwdCurveRates` As `Variant`
- `LongFwdCurveMat` As `Variant`
- `LongFwdCurveRates` As `Variant`

Report the VBA code in your solution.

1.3 Calibrating a swap market with multiple forward curves

Using the market data found in the spreadsheet, estimate the 3M and 6M forward curves as well as the discounting curve (all represented as zero coupon curves) for the EUR market using hermite interpolation. Since the market data contains money market basis swaps, you should use the function written in the above question.

You should use the knot points listed in the Data sheet. The listed CCS par spreads are quotes against 3M USD LIBOR flat. You should estimate the curves under the assumption that you are representing a bank that can fund itself at 3M USD LIBOR.

Report your estimated knot point rates with 4 digits (e.g. 4.0000%). Plot the 3M and 6M forward xIBOR rates (Act/360, MF) in 3M spacings for the next 30Y. Also, plot in a separate diagram the forward 3M CCS breaks (Act/360, MF) as well as forward starting money market basis swaps (of a 6M maturity using Act/360, MF)in the same 3M spacings.

Hint: If you cannot get the VBA function to work, you can still solve this question by pricing the money market basis swaps using the functions in the `fidAnalytics` library.

1.4 Terminating an IRS

Assuming that you have the swap listed in data sheet on your books, what is the mid-market PV of this swap on the curves calibrated above?

Assuming that you had been ignorant of both money market and cross currency basis, and had calibrated only a single zero coupon curve (used both for discounting and for calculating forward xIBOR rates) to the market quotes for the EUR IRSs, what would the PV then have been? What is the explanation for this difference in PV?

1.5 Calculating market rate delta vectors in the presence of money market basis

Using the Jacobian trick, report the market rate delta vector including money market basis swaps for the swap listed in the data sheet. Explain the intuition behind the swap's market rate delta vector.

1.6 Money market basis as two swaps

Instead of trading money market basis as a float-float swap, you can alternatively trade it as two fixed-for-float IRSs — one indexed against the shorter tenor and the other indexed against the longer dated tenor. Explain how this generates a money market basis position. Since the basis spread in the two-IRS construction is expressed as a difference between two plain vanilla fixed rates, calculate these par basis spreads for the EUR market for the following maturities: 1Y, 2Y, 5Y, 10Y, 15Y and 30Y. Report your findings as a number of basis points with two digits and explain why these basis spreads are slightly different from the ones you calibrated to.

2 The EUR cap/floor market

2.1 Calibrating a grid of SABR models

Using the listed zero coupon rates for the forward- and discount curves calibrate a grid of three SABR models to fit the EUR cap market. You should fix $\alpha = 52\%$ and then calibrate the three SABR models for the expiry knot points listed in the data sheet. For the interim expiries, you should interpolate between expiries of the three SABR models to find the relevant SABR parameters (use interpolation and interpolate between time-to-expiration using the Act/365 day count convention).

The market data are the premia for spot starting plain vanilla EUR caps listed in basis points of the notional. As such, they follow the conventions listed in the data sheet and all have their first fixing in 6M time.

Hint: Set up a table will all the relevant caplets (using `fidGenerateSchedule`) and price each cap as a sum of the relevant caplets. Remember to set the relevant constraints on the SABR parameters in `SOLVER` when calibrating the models.

2.2 Pricing a forward starting floor

Use the grid of SABR models calibrated above to price a 5X10 plain vanilla floor with a strike of 1.75% (that is, a floor that has its first fixing in 5Y time and covers the following 5Y period). What is the PV of such a contract on a notional of EUR 50,000,000?

2.3 SABR vs. Black'76

For the floor in question above, calculate the net forward- and discounting curve Dv01 (i.e. a single number) using both the SABR and Black'76 model. You should perform this calculation for each strike in the interval 0.25% to 5.00% in 25 bps increments. Plot a graph of your results and explain qualitatively why the two models yield different risk figures.

Hint: Remember that the Black'76 model gives the same price as the SABR model for plain vanilla floorlets if you just use the relevant Black'76 volatility that comes of out the SABR formula. It is only when you start to look at risk figures that the two models start to differ.

3 The CDS market

3.1 Calibrating a CDS curve

Use the quotes listed in the data sheet to calibrate a set of intensities. The market quotes are the upfront payments in basis points of the notional (to be paid in 2B, MF) corresponding to a running spread of 100 bps (quarterly Act/360, MF). Use the listed knot points and report your calibrated intensities with 4 digits (e.g. 4.0000%).

3.2 Price a forward starting CDS

Using the intensities calibrated above, what is the par spread (with no upfront payment) for a CDS with 6Y forward start and 4Y maturity on a notional of EUR 10,000,000?

3.3 The risk profile of a forward starting CDS

Assume that you have entered into the above forward starting CDS, buying protection at 100 bps. Investigate what happens to the PV of this contract as you increase the underlying intensities. Compare this to the PV of an identical contract that has spot (2B) rather than forward start. Plot the PVs of these two contracts using flat intensity curves for the range $\{0, \ldots, 25\%\}$ in 25 bps increments. The intensities should be shown on the x-axis and the PVs of the contracts should be shown on the y-axis.

Explain the two graphs and provide intuition as to why the two contracts behave differently.